# — : Bernoulli's equation : —

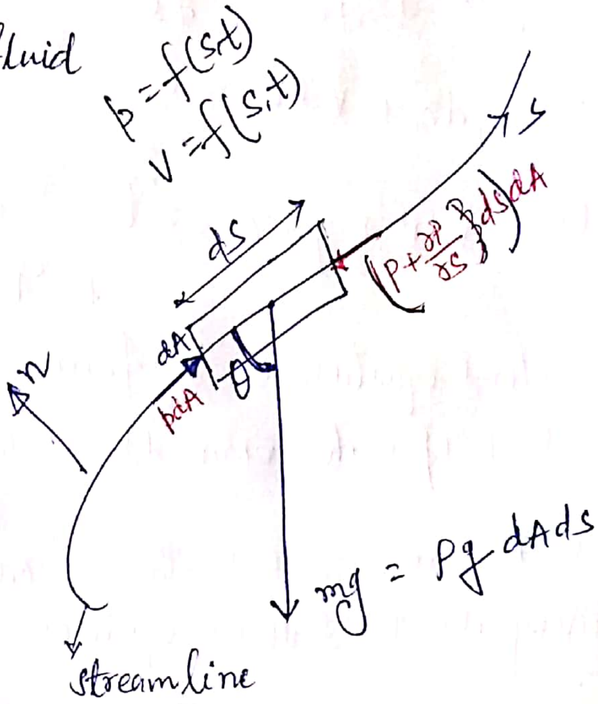
## * Euler equation of motion:

- It is the equation of motion in which forces due to gravity and pressure are taken into consideration.

Forces acting on the cylindrical fluid element:

$p = f(s,t)$
$v = f(s,t)$

1. pressure force $p\,dA$ in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of the fluid element $\rho g\, dA\, ds$.




Now, from Newton's 2nd law:

Net force in $s$ direction $=$ (mass of the fluid element) $\times$ (acceleration in $s$-direction)

$$\Rightarrow p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta = (\rho\, dA\, ds) \times a_s$$

Now, $a_s = \frac{Dv}{Dt} = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t}$

$$= v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

If the flow is steady, then $\frac{\partial v}{\partial t} = 0$. $\Rightarrow \boxed{a_s = v \frac{dv}{ds}}$

Now,

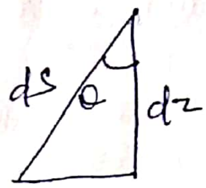
So, $p\,dA - \left(p + \frac{dp}{ds} ds\right) dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v \frac{dv}{ds}$

$$\Rightarrow -\frac{dp}{ds} \cancel{ds\,dA} - \rho g \cancel{dA\,ds} \cos\theta = \rho \cancel{dA\,ds} \times v \frac{dv}{ds}$$

$$\Rightarrow -\frac{dp}{ds} - \rho g \cos\theta = \rho\, v \frac{dv}{ds} \quad \text{①}$$

dividing by '$\rho$' both side and taking (-ve) common,

$$\frac{dp}{\rho\, ds} + g\cos\theta + v\frac{dv}{ds} = 0$$




$$\cos\theta = \frac{dz}{ds}$$

$$\Rightarrow \frac{dp}{\rho\, ds} + g\frac{dz}{ds} + v\frac{dv}{ds} = 0$$

$$\Rightarrow \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + v\frac{dv}{ds} = 0$$

$$\Rightarrow \frac{dp}{\rho} + g\,dz + v\,dv = 0$$

(or) $$\boxed{\frac{dp}{\rho} + v\,dv + g\,dz = 0}$$ This equation is known as Euler's equation.

Note: (1) Euler's equation is a force equation.

~~(2) Unit of each term is Newton/kg~~

* Now, work done by force can be calculated by integrating F.ds.
So, integrating Euler's equation we get Bernoulli's eqn.

## Bernoulli's eqn from Euler's equation

By integrating the Euler's equation,

$$\int\frac{dp}{\rho} + \int v\,dv + \int g\,dz = \text{Constant}$$

If flow is incompressible, $\rho$ is constant

So, $$\boxed{\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{Constant}}$$ ⇒ ~~unit~~ ⇒ energy per unit mass

dividing both side by 'g'

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}}$$ Bernoulli's equation

↳ valid along a stream line.

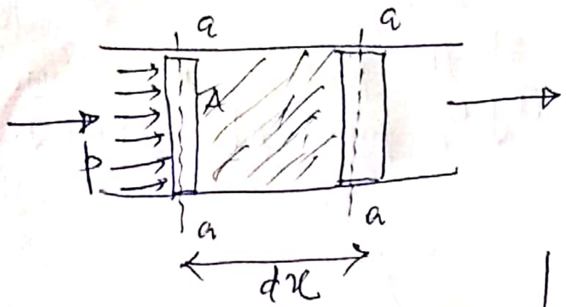
$\frac{p}{\rho g}$ → Press$^r$ energy per unit weight (or) press$^r$ head

$\frac{v^2}{2g}$ → Kinetic energy per unit mass (or) Kinetic head (or) velocity head

$z$ → Potential energy per unit weight (or) Potential head (or) Datum head

Additional concept

**Note:** Is $\frac{p}{\rho g}$ equals to press^r energy per unit weight?




Force $= p \cdot A$

Work done $= F \cdot dx = pA\,dx$

$$\frac{\text{Work}}{\text{mass}} = \frac{pA\,\cancel{dx}}{\rho\,\cancel{A\,dx}} = p/\rho$$

So, $$\frac{\text{Work}}{\text{weight}} = \frac{p}{\rho \times g}$$

So, $\frac{p}{\rho g}$ is called flow work per unit wt. or press^r energy per unit wt.

- A fluid element has to possess $\frac{p}{\rho g}$ amount of press^r energy to ~~reach from~~ push the adjacent next layer and reach from 1st to 2nd position.

- piezometric head = Press^r head + Datum head.
- So, $$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2}$$

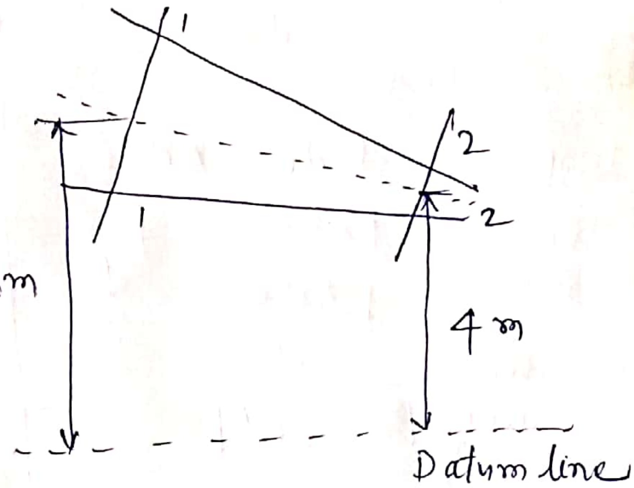
*Assumptions made in derivation ~~of~~ Bernoulli's eqn: -

1. The flow is steady
2. The fluid is ideal means, viscosity is zero.
3. The flow is incompressible, mean $\boxed{\rho = \text{constant}} \Rightarrow \boxed{d\rho = 0}$
4. Gravity is a only body force.
5. The flow is irrotational.

**Statement:** A/c to Bernoulli's equation, the total mechanical energy i.e. sum of K.E, Press^r energy and P.E. along a stream line is same for a steady, ideal flow of an incompressible fluid where gravity is the only ~~B~~ body force.

Ex: The water is flowing through a pipe having diameter 20 cm and 10 cm at sections 1 & 2 respectively. The rate of flow through pipe is 35 litre/sec. The section 1 is 6 m above datum, and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.

Sol$^n$




**At section 1,**

$D_1 = 20\text{ cm} = 0.2\text{ m}.$

$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.2)^2 = 0.314\text{ m}^2$

$P_1 = 39.24\text{ N/cm}^2$

$= 39.24 \times 10^4\text{ N/m}^2$

$Z_1 = 6\text{ m}.$

**At section 2.**

$D_2 = 0.10\text{ m}$

$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785\text{ m}^2$

$Z_2 = 4\text{ m}.$

$P_2 = ?$

Rate of flow $(Q) = 35$ litre/s

$= \frac{35}{1000}\text{ m}^3/\text{s} = 0.035\text{ m}^3/\text{s}$

$Q = A_1 V_1 = A_2 V_2$

$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.314} = 1.114\text{ m/s}$

$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456\text{ m/s}.$

Now, applying Bernoulli's eqn at section 1 & 2.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$$\Rightarrow 40 + 0.063 + 6.0 = \frac{P_2}{9810} + 1.012 + 4.0$$

$$\Rightarrow 46.063 = \frac{P_2}{9810} + 5.012$$

$$\Rightarrow P_2 = 41.051 \times 9810\text{ N/m}^2 = 402710.31\text{ N/m}^2$$

$$= \boxed{40.27\text{ N/cm}^2} \text{ A}$$

# Bernoulli's equation for real fluid

- One of the assumptions made in derivation of Bernoulli's theorem is that flow should be ideal, means viscosity is zero.
- But real fluids have some viscosity and hence offer some resistance to flow. Thus there are always some losses in fluid flow. And hence these losses have to be taken into consideration.
- Thus Bernoulli's equation for real fluids b/w section 1 and 2 is given as —

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L}$$

→ Head loss

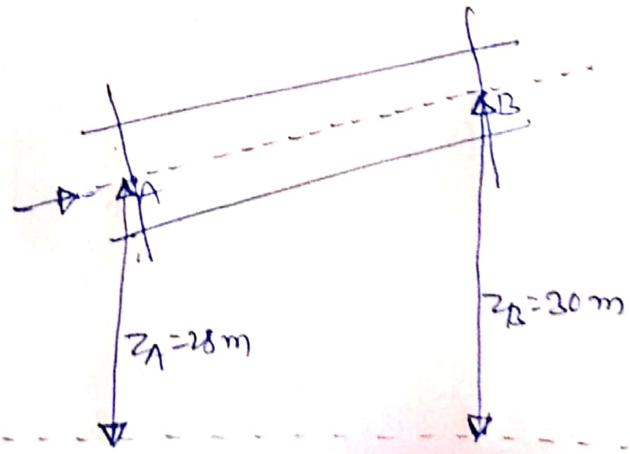
where, $h_L$ is the loss of energy between points 1 and 2.

**Ex** A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head ~~at A a~~ b/w A and B.

**Soln**

$D = 400mm = 0.4 m$
$v = 25 m/s$

$p_A = 29.43\ N/cm^2 = 29.43 \times 10^4\ N/m^2$
$z_A = 28\ m.$
$v_A = v = 25 m/s$




Total mechanical energy at A.

$$E_A = \frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28 = 89.85\ m$$

At point B,

$$p_B = 22.563\ N/cm^2 = 22.563 \times 10^4\ N/m^2$$

$$Z_B = 30\ m$$

$$V_B = V = V_A = 25\ m/s$$

Total energy at B,

$$E_B = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + Z_B$$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = \underline{84.85\ m.}$$

So, Loss of energy $(h_L)$ = (Total Mech. energy at A) − (Total mech. energy at B)

$$= 89.85\ m - 84.85\ m = \boxed{5.0\ m.}$$ Ans

→ loss of head.